

## Description of Subarea III – Mathematics

The skills addressed in this subarea require demonstration of quantitative literacy at the college level through the application of mathematical methods and reasoning to the solution of real-world problems.

### Standard 0015 –

Apply mathematical reasoning skills to analyze patterns and solve problems.

#### Examples

- Draw conclusions using inductive reasoning.
- Draw conclusions using deductive reasoning.
- Identify errors in mathematical explanations.

#### Suggested strategies & key vocabulary needed

##### Inductive reasoning

- Definition – Inductive reasoning is from a specific case or cases and deriving a general rule. It draws inferences from observations in order to make generalizations.
- Inference can be done in four stages:
  - *Observation*: collect facts, without bias.
  - *Analysis*: classify the facts, identifying patterns of regularity.
  - *Inference*: From the patterns, infer generalizations about the relations between the facts.
  - *Confirmation*: Testing the inference through further observation.
- Inductive reasoning strategies
  - Derive a general rule in an accepted area and then apply the rule in the area where you want the situation or person to behave.
  - Give lots of detail, then explain what it all means.
  - Talk about the benefits of the parts and only get to the overall benefits later.
  - Take what has happened and give a plausible explanation for why it has happened.

##### Deductive reasoning

- Definition – Deductive reasoning starts with a general case and deduces specific instances. Deductive reasoning starts with an assumed hypothesis or theory.
- Deductive reasoning assumes that the basic law from which an argument is made is applicable in *all* cases. This can let you take a rule and apply it perhaps where it was not really meant to be applied.
- Deductive conclusions can be *valid* or *invalid*.

#### Websites for additional help

- Basic Math Help - <http://www.basic-mathematics.com>
- ACT practice test – math section - [http://www.actstudent.org/sampletest/math/math\\_01.html](http://www.actstudent.org/sampletest/math/math_01.html)
- ACT practice test booklet p. 17-20 (can be printed) (answers at the end of p. 20) <http://sat.collegeboard.org/SAT/public/pdf/getting-ready-for-the-sat-subj-tests.pdf>
- ACT practice test booklet explanations for test questions <http://sat.collegeboard.org/SAT/public/pdf/SubjectTestsAnswerExplanationsMath.pdf>

### Practice test samples

#### Examples of inductive reasoning practice items

Examples of inductive reasoning are numerous. Lots of IQ or intelligence tests are based on inductive reasoning. Patterns and inductive reasoning are closely related.

Find here a couple of good examples of inductive reasoning that will really help you understand inductive reasoning

But what is inductive reasoning?

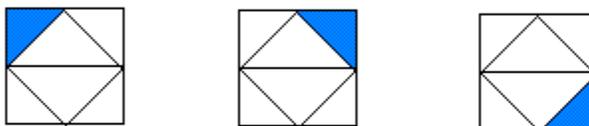
Inductive reasoning is making conclusions based on patterns you observe.

The conclusion you reach is called a conjecture

#### Shapes and inductive reasoning:

##### Example #1:

Look carefully at the following figures. Then, use inductive reasoning to make a conjecture about the next figure in the pattern



If you have carefully observed the pattern, may be you came up with the figure below:



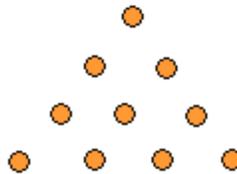
**Example #2:**

Look at the patterns below. Can you draw the next figure or next set of dots using inductive reasoning?



The trick is to see that one dot is always placed between and above two dots. Also, the next figure always has one more dot at the very bottom row

keeping this in mind, your next figure should look like this:



*Other examples of inductive reasoning: Integers and inductive reasoning*

**Example #3:**

Take a look at this table that shows multiplication as repeated addition:

<b>Multiplication</b>	<b>Repeated addition</b>	<b>Sum</b>
$4 \times -2$	$-2 + -2 + -2 + -2$	- 8
$3 \times -7$	$-7 + -7 + -7$	- 21
$5 \times -6$	$-6 + -6 + -6 + -6 + -6$	- 30

What do you notice about the signs of the sums?

Since the sum is always negative, the pattern suggests that the product of a positive integer and a negative integer is negative

**Example #4:**

Look at the following patterns:

$$3 \times -4 = -12$$

$$2 \times -4 = -8$$

$$1 \times -4 = -4$$

$$0 \times -4 = 0$$

$$-1 \times -4 = 4$$

$$-2 \times -4 = 8$$

$$-3 \times -4 = 12$$

Every time the factor on the left is decreased by 1, the answer is increased by 4

However, the pattern suggests that a negative times a negative is a positive

### **Examples of deductive reasoning practice items**

Example 1 - deductive argument:

1. All men are mortal
2. Socrates is a man
3. Therefore, Socrates is mortal

The first premise states that all objects classified as "men" have the attribute "mortal". The second premise states that "Socrates" is classified as a man – a member of the set "men". The conclusion states that "Socrates" must be mortal because he inherits this attribute from his classification as a man.

Example 2 - The law of [syllogism](#) takes two conditional statements and forms a conclusion by combining the hypothesis of one statement with the conclusion of another. The following is an example:

1. If Larry is sick, then he will be absent from school.
2. If Larry is absent, then he will miss his classwork.
3. If Larry is sick, then he will miss his classwork.

We deduced the solution by combining the hypothesis of the first problem with the conclusion of the second statement. We also conclude that this could be a false statement.

Example 3 - The following is an example of an deductive argument that is valid, but not sound; a premise is false:

1. Everyone who eats steak is a quarterback.
2. John eats steak.
3. Therefore, John is a quarterback.

The example's first premise is false (there are people who eat steak that are not quarterbacks), but the conclusion must be true, so long as the premises are true (i.e. it is impossible for the premises to be true and the conclusion false). Therefore the argument is valid, but not sound.

Example 4

All apples are fruit.  
All fruits grow on trees.  
Therefore all apples grow on trees.

Or

All apples are fruit.  
Some apples are red.  
Therefore some fruit is red.

Example 5 – differences between inductive and deductive reasoning

Ex of deductive:

1. All men are mortal. (given)
2. Socrates was a man. (given)
3. Socrates was mortal. (conclusion)

In deductive, you're just drawing conclusions from known facts. If the given facts are true, then there's no way that your conclusion is wrong. You don't really get any "new" knowledge from deductive reasoning; you just reorganize old knowledge.

Ex of inductive:

1. Socrates was Greek. (given)
2. Most Greeks ate fish. (given)
3. Socrates probably ate fish. (conclusion)

Inductive reasoning draws conclusions that are most likely not wrong. However, your conclusion is not necessarily right - maybe Socrates was allergic to fish. Inductive reasoning give new knowledge, with the tradeoff that your conclusion may be faulty.

Example 6 – more sample inductive and deductive reasoning

Examples of Deductive Arguments

All dogs are mammals.  
All mammals have kidneys.  
Therefore all dogs have kidneys.

Since all squares are rectangles,  
and all rectangles have four sides,  
all squares have four sides.

All chemists are smart,  
since chemists are scientists and all scientists are smart.

(Note: Although the conclusion is probably false, the flaw in the argument is that one of the premises is presumably false; it remains true that if the premises are true, the conclusion must also be true.)

Since all men are mortal,  
and Socrates is a man,  
Socrates is mortal.

The sun is a star;  
the sun has planets;  
therefore some stars have planets.

#### Examples of Inductive Arguments

All swans we have seen have been white;  
therefore all swans are white.

All swans we have seen have been white; therefore the next swan we see will be white.

All known planets travel about the sun in elliptical orbits;  
therefore all planets travel about the sun in elliptical orbits.

Exploration of the surface of Mars has produced some surprising facts.  
Therefore exploration of the surface of Jupiter will produce some surprising facts.

Since Chris is a good athlete,  
Chris's sister must be a good athlete also.

#### Resources for intervention

- Basic math test to see where you have deficits - <http://www.basic-mathematics.com/basic-math-test.html>
- Deductive reasoning videos
  - <http://www.askkids.com/resource/Deductive-Reasoning-Examples.html>

- <http://www.khanacademy.org/video/deductive-reasoning-1?playlist=Algebra+I+Worked+Examples>
- <http://www.khanacademy.org/video/deductive-reasoning-2?playlist=Algebra+I+Worked+Examples>
- <http://www.khanacademy.org/video/deductive-reasoning-3?playlist=Algebra+I+Worked+Examples>
- Inductive reasoning videos
  - <http://www.khanacademy.org/video/u12-11-t3-we1-inductive-reasoning-1?playlist=Algebra+I+Worked+Examples>
  - <http://www.khanacademy.org/video/inductive-reasoning-2?playlist=Algebra+I+Worked+Examples>
  - <http://www.khanacademy.org/video/inductive-reasoning-3?playlist=Algebra+I+Worked+Examples>